EXERCISE – V

JEE PROBLEMS

1. If in the expansion of $(1 + x)^m (1 - x)^n$, the co-efficients of x and x^2 are 3 and – 6 respectively, then m is **[JEE 99,2]**

(A) 6 (B) 9 (C) 12 (D) 24

Sol.

2. For $2 \le r \le n$, $\binom{n}{r} + 2\binom{n}{r-1} + \binom{n}{r-2} =$

(A) $\binom{n+1}{r-1}$ (B) $2\binom{n+1}{r+1}$ (C) $2\binom{n+2}{r}$ (D) $\binom{n+2}{r}$

Sol.

3. For any positive integers m, n (with $n \ge m$), let $\binom{n}{m} = {}^nC_m$. Prove that $\binom{n}{m} + \binom{n-1}{m} + \binom{n-2}{m} + \ldots + \binom{m}{m} = \binom{n+1}{m+1}$

Hence or otherwise prove that,

$$\binom{n}{m} + 2\binom{n-1}{m} + 3\binom{n-2}{m} + \ldots + (n-m+1)\binom{m}{m} = \binom{n+2}{m+2}$$

[JEE 2000 (Mains), 6]

Sol.

4. Find the largest co-efficient in the expansion of $(1 + x)^n$, given that the sum of co-efficients of the terms in its expansion is 4096. **[REE 2000 (Mains)]**

Sol.

5. In the binomial expansion of $(a - b)^n$, $n \geq 5$, the sum of the 5th and 6th terms is zero. Then $\frac{a}{b}$ equals.

[JEE 2001 (Scr.), 3]

(A) $\frac{n-5}{6}$ (B) $\frac{n-4}{5}$ (C) $\frac{5}{n-4}$ (D) $\frac{6}{n-5}$

Sol.

6. Find the coefficient of x^{49} in the polynomial

[REE 2001 (Mains), 3]

$$\left(x-\frac{C_1}{C_0}\right)\left(x-2^2\frac{C_2}{C_1}\right)\left(x-3^2.\frac{C_3}{C_2}\right)..............\left(x-50^2,\frac{C_{50}}{C_{49}}\right)$$

where $C_r = {}^{50}C_r$

Sol.

7. The sum $\sum_{i=0}^{m}\binom{10}{i}\binom{20}{m-i}$, (where $\binom{p}{q}$ = O if P < q) is maximum when m is **[JEE 2002 (Scr.), 3]**

(A) 5 (B) 10 (C) 15 (D) 20

Sol.

8. (a) Coefficient of t^{24} in the expansion of $(1 + t^2)^{12} (1 + t^{12}) (1 + t^{24})$ is **[JEE 2003 (Scr.), 3]**

(A) ${}^{12}C_6 + 2$ (B) ${}^{12}C_6 + 1$ (C) ${}^{12}C_6$ (D) none

Sol.

(b) Prove that :

$$2^k \cdot \binom{n}{0}\binom{n}{k} - 2^{k-1}\binom{n}{1}\binom{n-1}{k-1} + 2^{k-2}\binom{n}{2}\binom{n-2}{k-2} \ldots\ldots\ldots (-1)^k \binom{n}{k}\binom{n-k}{0} = \binom{n}{k}$$

[JEE 2003 (Mains),2]

Sol.

9. ${}^{n-1}C_r = (k^2 - 3).\ {}^{n}C_{r+1}$, if $k \in$ **[JEE 2004 (Scr.)]**

(A) $[-\sqrt{3}, \sqrt{3}]$ (B) $(-\infty, -2)$ (C) $(2, \infty)$ (D) $(\sqrt{3}, 2]$

Sol.

10. The value of

$$\binom{30}{0}\binom{30}{10} - \binom{30}{1}\binom{30}{11} + \binom{30}{2}\binom{30}{12} + \ldots\ldots\ldots + \binom{30}{20}\binom{30}{30} \text{ is,}$$

where $\binom{n}{r} = {}^{n}C_r$ **[JEE 2005 (Scr.)]**

(A) $\binom{30}{10}$ (B) $\binom{30}{15}$ (C) $\binom{60}{30}$ (D) $\binom{31}{10}$

Sol.

11. The number of seven digit integers, with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only, is **[JEE 2009]**

(A) 55 (B) 66 (C) 77 (D) 88

Sol.

12. For r = 0, 1, ..., 10 let A_r, B_r, C_r denote, respectively, the coefficient of x^r in the expansions of $(1 + x)^{10}$, $(1 + x)^{20}$ and $(1 + x)^{30}$. Then $\sum_{r=1}^{10} A_r(B_{10}B_r - C_{10}A_r)$ is equal to **[JEE 2010]**

(A) $B_{10} - C_{10}$ (B) $A_{10}\left(B_{10}^2 - C_{10}A_{10}\right)$

(C) 0 (D) $C_{10} - B_{10}$

Sol.

Paragraph for Question Nos. 13 to 14

Let a_n denote the number of all n-digit positive integers formed by the digits 0, 1 or both such that no consecutive digits in them are 0. Let b_n = the number of such n-digit integers ending with digit 1 and c_n = the number of such n-digit integers ending with digit 0. **[JEE 2012]**

13. Which of the following is correct ?

(A) $a_{17} = a_{16} + a_{15}$ (B) $c_{17} \neq c_{16} + c_{15}$

(C) $b_{17} \neq b_{16} + c_{16}$ (D) $a_{17} = c_{17} + b_{16}$

Sol.

14. The value of b_6 is

(A) 7 (B) 8 (C) 9 (D) 11

Sol.